Fatigue Damage Modelling Techniques for Textile Composites: Review and Comparison with UD Composite Modelling Techniques

R.D.B. Sevenois *

MSc, Department of Materials Science and Engineering
Faculty of Engineering and Architecture
Ghent University
Technologiepark Zwijnaarde 903
B-9052 Zwijnaarde
Belgium
Email: ruben.sevenois@ugent.be

MSc, SIM program M3Strength
Technologiepark Zwijnaarde 935
B-9052 Zwijnaarde
Belgium

W. Van Paepegem
Prof., Department of Materials Science and Engineering
Faculty of Engineering and Architecture
Ghent University
Technologiepark Zwijnaarde 903
B-9052 Zwijnaarde
Belgium

ABSTRACT

Composite structural parts have been successfully introduced in high performance industries. Nowadays, also lower performance, high volume production industries are looking for the application of composites in their products. Especially attractive are textile composites (woven, braided, etc..) because of their better drapability and higher resistance to out-of-plane and dynamic loads. Currently, however, extensive mechanical tests are needed to properly design a composite structure. This is a requirement the large volume industries typically do not have the resources nor the time for. Reducing the need for structural tests can only be done if reliable simulation techniques are available. Simulation techniques for fatigue loading are particularly interesting because products generally have to perform their function over a period of time. For the textile structural composites concerned in this paper, some notable modelling techniques have been developed over the past 15 years. These techniques are presented here and the state of the art is established together with insights for future development by comparing the state of the art with the modelling techniques for laminates from unidirectional laminae.

1 Introduction

Since the promises of weight loss, lower fuel consumption and increased performance have been delivered by the use of unidirectional (UD) composite laminates in high performance industries (e.g. aerospace, sports, etc...), other industries (e.g. automotive, construction, etc...) are also looking at this type of materials. Their interest follows, among other things, from ecological requirements to be imposed in the near future by governments and the growing interest of

---

[*] Corresponding Author

the general public. Less demanding industries, however, are often involved in the production of highly curved products produced in large volumes for which UD laminates seem to be less suitable. This gap could be filled with the use of Fibre Reinforced Polymers (FRP) with woven, braided or other three-dimensional fibre architectures which have better drapability and higher resistance to out-of-plane and dynamic loads. Additionally, the use of a thermoplastic (instead of the common thermoset) matrix should be more suitable for high volume production [1].

Next to high volume production, the markets of these industries are more competitive due to the greater amount of manufacturers and the closer relation to the general public. Therefore, they must be able to adapt quickly, requiring a faster time to market at the lowest possible cost. There is therefore no room for extensive mechanical tests, which are currently the standard in the high performance markets [2], while the reliability of the product cannot be sacrificed. One thus has to turn to modelling techniques to predict the product properties. Particularly interesting are fatigue damage modelling techniques since products are often subjected to repeated loading. The complicated behaviour of FRP, as compared to metals [3, 4], hinders the development of these models.

Research on fatigue damage modelling for FRP has been going on for many years [5, 6] and the scientific community has been able to identify the most significant mechanisms behind fracture and fatigue (e.g. microcracking, delamination, etc..). The behaviour of these mechanisms (e.g. why do cracks grow in a specific way and speed when subjected to a certain loading), however, is not yet well understood [6]. This forces researchers in the field of modelling to use relationships derived from experimental observations to predict the behaviour of a composite structure. Even the well established Paris Law for crack growth falls within this category [6]. As such, every modelling theory is validated for a specific purpose only. Not surprisingly, most attempts to compare several theories to a common experimental dataset have concluded that none is superior to another and their accuracy is greatly dependent on the individual cases [7, 8]. These conclusions keep the door open for the simultaneous existence of multiple theories, a phenomenon also observed in the area of quasi-static strength prediction [9]. The techniques for fatigue damage modelling of FRP with woven, braided and three-dimensional fibre architectures in general have expanded significantly in the past 15 years. In addition, the interest of industry in these structures is increasing. Since no review paper on this topic has been published yet, it is appropriate to review the developments and establish a state of the art.

The purpose of this paper is to give an overview of the existing techniques for fatigue damage modelling of FRPs with woven, braided and other three-dimensional fibre architectures. A classification proposed by Degrieck and Van Paepegem in 2001 [5] is modified to accommodate developing research areas and changes in terminology. A total of four categories is considered: (i) “Fatigue Life Models”, (ii) “Residual Strength Models”, (iii) “Residual Stiffness Models” and (iv) “Mechanistic Models”. The overview of existing models serves as a basis for a comparison to the recent models used for FRPs from UD laminae. In this discussion similarities and difficulties are identified. The conclusions of this discussion will be used as a foundation for novel research in this area.

In the following section, first a brief overview of the fatigue failure process for composites from textile laminates is presented. Next, a justification is provided for the classification of the fatigue modelling techniques, Section 3,followed by the discussion of the fatigue modelling techniques, Section 4 to Section 7, in their aforementioned order. Section 8 contains the comparison with modelling techniques for UD FRP.

Although this work does not pretend to be exhaustive, the most important theories and fundamental equations on which various models are based are presented in this paper. Section 3 is applicable to fatigue damage modelling techniques for FRP in general. From Section 4, this review is strictly applicable to FRP with textile fibre architectures. For reviews on modelling techniques and fatigue damage models for UD FRP, the reader is referred to [5, 10, 11].

2 Typical Failure Process of Textile Composites Subjected to Fatigue Loading

From the first experiments on modern composite structures, it was quickly identified that the failure mechanisms for structures made from this type of material are far more complicated than their metal counterparts. Over the years, the failure mechanisms which occur during mechanical loading of a structure have been thoroughly studied and nowadays, the most important ones are known. Because a basic knowledge on the general failure process of textile composite laminates subjected to tension and compression fatigue will help to understand the main body of this paper, the failure process is briefly presented in this Section. For this, the works by Daggumati et al. [12] and Cox et al. [13] have been used as a guideline.

In contrast to the fibres in UD composite materials, the fibre bundles in textile composites are woven, braided or knitted together into a specific pattern. Because of this, the fibre bundles are not straight but are positioned in a three-dimensional fibre architecture. During the consolidation of multiple textile plies, the fibre architecture of the plies is usually somewhat distorted due to nesting which results in a similar microscopic cross section as is shown in Figure 1.

Considering failure due to tension fatigue, damage starts early in the fatigue process with cracking of the weft yarns perpendicular to the loading direction, Figure 1a. Continuing loading, these microcracks grow and cause breakage of axial fibres and meta-delaminations, Figure 1b, but stay contained within their respective plies. Growing further, the contained cracks join across the ply boundaries to form large cracks, Figure 1c. Eventually, in the final stages of the com-

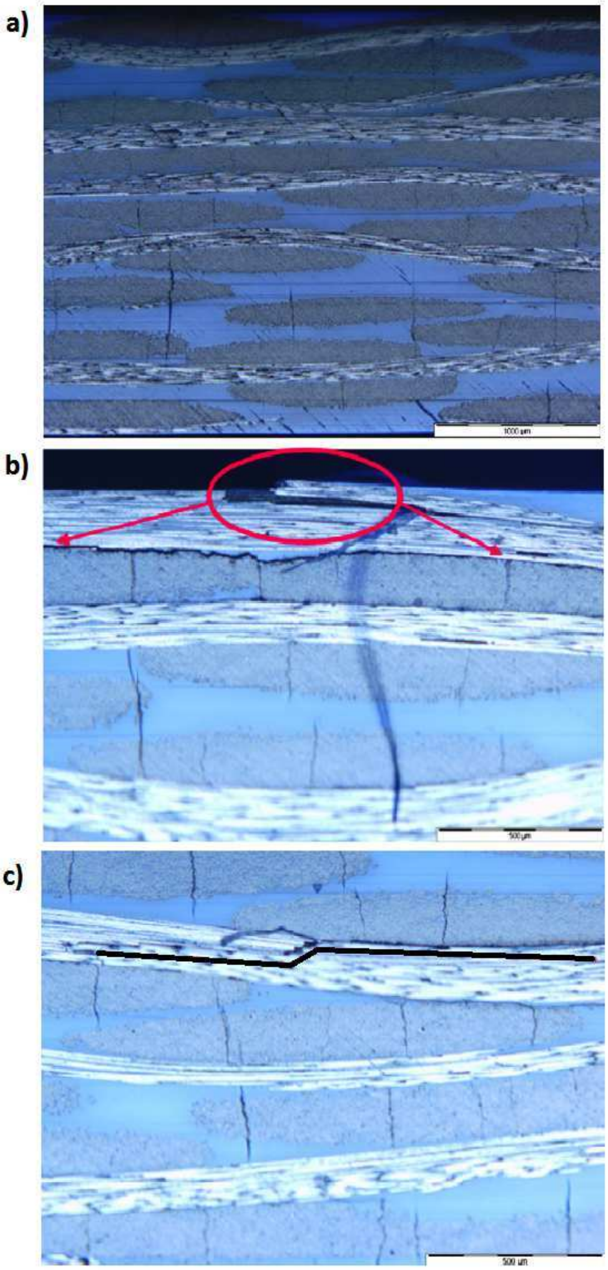

Fig. 1. Daggumati et al. [12], microscopic investigation of the polished edges of a carbon-PPS satin weave. a) weft yarn damage; b) broken axial fibres and meta-delamination; c) crack conjunction

ponent life, interply delamination occurs together with a large amount of broken axial fibres and meta-delaminations, Figure 2a-b.

The aforementioned failure mechanisms are not significantly different from the mechanisms which occur upon compressive loading, Figure 3, except for the addition of yarn buckling and fibre kinking, Figure 4. As in the tensile case, failure initiation usually starts with microcracks which grow further to meta-delaminations. These meta-delaminations, however, destabilize the fibre bundles and diminishes their resistance against buckling. Depending on the location of the fibre bundle within the composite laminate, bundles will either buckle or kink. If the bundle is not restrained by other, intact, fibre bundles, for example when it lies close to one of the outer edges of the structure, the fibre bundle will buckle. If it is sufficiently restrained to deform out-of-plane, for example when the bundle is located in the most inner plies of the laminate, the bundle is more prone to kink as is shown in Figure 4.

The two additional failure mechanisms tend to be quite detrimental for the load carrying capability of the structure since the associated out-of-plane deformation induces high peel forces at the bundle edges. This accelerates the damage

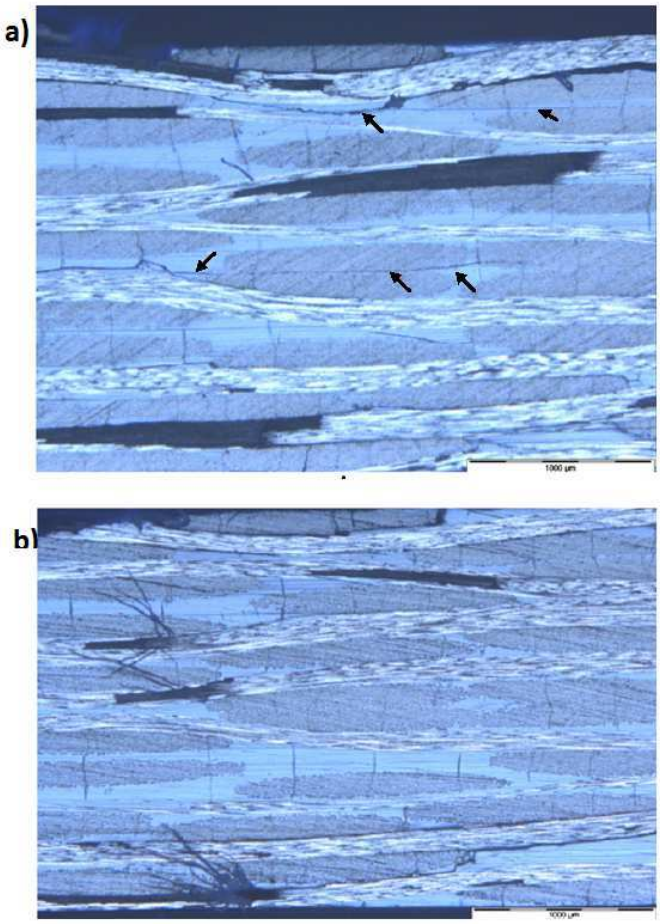

Fig. 2. Daggumati et al. [12], microscopic investigation of the polished edges of a carbon-PPS satin weave. a) inter-ply delamination; (b broken axial fibres and meta-delamination

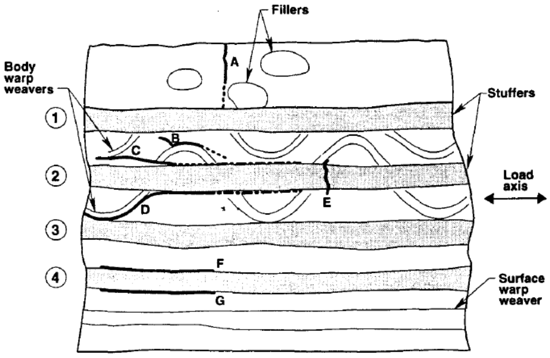

Fig. 3. Cox et al. [13], schematics of failure events in angle interlock specimens under monotonic compression

growth and usually results in much shorter fatigue life when compared to the case of fatigue under tension loading.

The discussion above concerned the failure mechanisms in pure tensile or compressive cyclic loading. A real structure, however, is usually subjected to a combination of these loads and shear over the span of its lifetime. Although this was not discussed in the previous paragraphs, the described mechanisms which occur during variable amplitude loading are a combination of the aforementioned, depending on which instantaneous load is applied at a certain time.

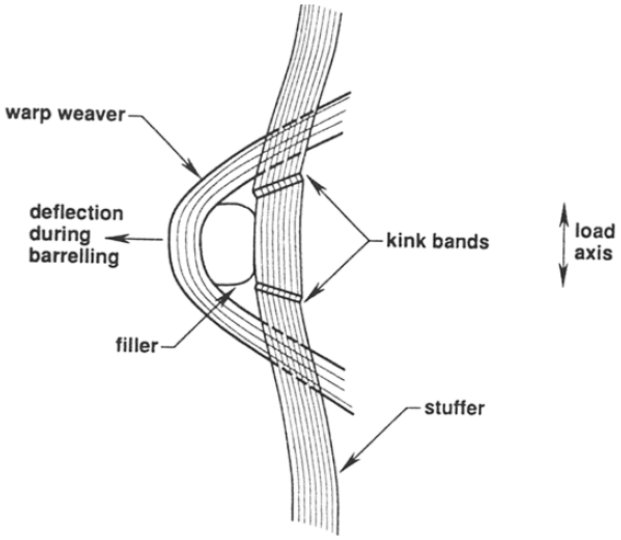

Fig. 4. Cox et al. [13], the formation of kink bands where a filler has been driven against a stuffer by a constraining warp weaver

3 Justification of Classification

A common use of terminology is of paramount importance in means of communication. Therefore, a clear definition of the classification for fatigue damage modelling techniques for FRP is established first. In this work, the original classification by Degrieck and Van Paepegem [5] is slightly modified to current state of the art. The classification is not posed as strict but, merely as, a well constructed guideline. The adaptations made and definitions are briefly given below. Should it be of interest to the reader, a historical perspective on the background of the choice for the modifications can be found in the Appendix.

Degrieck and Van Paepegem [5] divided the modelling techniques into: (i) fatigue life models, (ii) phenomenological models for residual stiffness/strength and (iii) progressive damage models. Two modifications, the reasons for which are the changed usage of the terminology and the more pronounced existence of Mechanistic modelling strategies, are proposed. First, since the term "progressive" can be interpreted in multiple ways, it is replaced with "Mechanistic". This way the modelling approach contained in this class is made more clear. Second, the "Residual Strength" and "Residual Stiffness" models, which were categorized under "phenomenological" model are separated and the term "phenomenological" is omitted. This results in the classes "Residual Strength Models" and "Residual Stiffness Models". The category "Fatigue Life Models" remains unchanged.

In conclusion, the proposed classification has four categories. The (i) "Fatigue Life Models", which attempt to predict the fatigue life of the structure under a certain loading spectrum, the laminate configuration and amount of cycles. These models are based on experimental data such as S-N curves and/or Goodman diagrams. (ii) "Residual Strength Models" and (iii) "Residual Stiffness Models", which predict the change in strength/stiffness during loading. These modelling approaches attempt to be less data dependent than the "Fatigue Life Models" and can also be used to take into account a change of stress state over time. (iv) "Mechanistic Models", where the fatigue laws attempt to model the actual mechanisms occurring during fatigue loading. These mechanisms include, for example, initiation and propagation of microcracks and delaminations.

4 Fatigue Life Models

"Fatigue Life Models" predict the (remaining) fatigue life of a component. They are mostly formulated using logaritmic curves with some fitting parameters. The variables used in these models commonly are: the applied amount of cycles, stress ratio, stress amplitude and/or lamina(te) stiffness and ply layup. The fitting parameters in the life and degradation laws are derived from experimental observations.

Some of the main contributors to the life prediction methods for laminates from woven plies are Kawai et al. [14,15]. Built upon previous experimental investigations [16] and experience with UD carbon/epoxy laminates [17] they developed the "anisomorphic Constant Fatigue Life (CFL) diagram approach". This approach "allows to construct the asymmetric and nonlinear CFL diagram for a given composite at a given temperature on the basis of the static strengths in tension and compression and the reference S-N relationship for the critical stress ratio" [14] for woven carbon/epoxy laminates. In the approach, for a certain composite laminate, the critical stress ratio, defined as the ratio between compressive and tensile static strength $\chi = \sigma_C/\sigma_T$, is used to divide the well-known constant life diagram, Figure 5, in 2 parts,

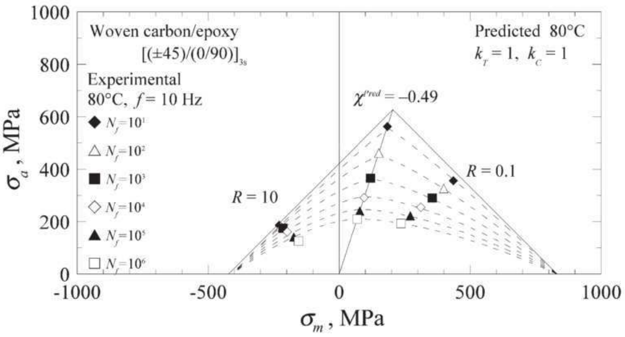

Fig. 5. Kawai et al. [15], example of a constant life diagram

a tension and compression dominated segment. To describe each segment the function shown in Eq. (1) is used.

$$-\frac{\sigma_a-\sigma_a^{(\chi)}}{\sigma_a^{(\chi)}}=\left(\frac{\sigma_m-\sigma_m^{(\chi)}}{\sigma_T\sigma_m^{(\chi)}}\right)^M \tag{1a}$$

$$M=2-\left(\psi_\chi\right)^k \tag{1b}$$

In this equation σ_a, σ_m and σ_T are the stress amplitude, mean stress and critical tensile stress respectively. ψ_χ is the fatigue strength ratio, which defines the shape of the constant life diagram, χ is the critical stress ratio and k adjusts the rate of change of the CFL curves from a straight line to a parabola. k and χ are different depending on whether one is in the compression or tension segment. ψ_χ can then be used to determine the material constants K_χ, a, b and n to the normalized S-N curve at the critical stress ratio:

$$N_f=\frac{1}{K_\chi}\frac{1}{\left(\psi_\chi\right)^n}\frac{\langle 1-\psi_\chi\rangle^a}{\langle\psi_\chi-\psi_{\chi^{(L)}}\rangle^b} \tag{2}$$

The method was validated for Quasi-Isotropic (QI) laminates from woven carbon/epoxy laminae. In later work [15], the method was generalized and the influence of temperature was added by considering the statistical effect of temperature on the aforementioned input parameters.

A simpler approach was taken by Vania and Carvelli [18]. By fitting a sigmoïdal like function to experimental fatigue life data for a single-ply 3D non-crimp orthogonal weave E-glass/epoxy and standard plain weave E-glass/epoxy [19], they were able to minimize the amount of required experimental data. For good prediction of a Wöhler-like curve, it appears that the used Sigmoïdal like function meets the expectations.

A similar approach was proposed by Mouritz [20] and Toumi et al. [21]. Mouritz assumed a simple log-linear relation between the amount of cycles N and applied stress level S for the fatigue life prediction of three-dimensional (z-pinned reinforced) FRP. The relation was also enhanced with the addition of a scaling factor σ_{3D}/σ_{2D} in an attempt to incorporate the effect of the three-dimensional reinforcement. In contrast, Toumi et al. [21] used a power law for the life prediction of woven E-glass/epoxy laminates. This choice was made specifically for the prediction of high cycle fatigue life. After application to experimental data, both authors concluded that their life prediction methods can be used for their chosen material systems respectively.

Tamuzs et al. [22, 23, 24] devised a way to construct a Master S-N curve for the prediction of fatigue life for off-axis loaded woven composites. Experimentally, two stress ratios, $R = 0.1$ [23, 24] and $R = -1$ [22], were investigated for multiple load angles of a 4-harness satin woven glass/epoxy laminate. They found that the Master S-N curve (which can be used to estimate S-N curves for other loading angles) could be obtained by either directly using experimental data or from the monitoring of damage accumulation.

Naderi and Khonsari [25, 26, 27, 28] investigated the possibility of a fatigue fracture criterion based on thermodynamic principles. They proposed to use the concept of Fracture Fatigue Entropy (FFE), derived from earlier work in metallic materials [29], to predict fatigue life. FFE is defined as the maximum value of the accumulated entropy which can be taken up by the material before failure. Thus, independent of frequency, load and specimen size, when the accumulated entropy during cycling reaches the FFE, the specimen fails. The accumulated entropy production can be calculated using Equation (3).

$$\gamma_f = \int_0^{t_f} \left(\frac{W_p}{T} - \frac{\mathbf{J}_q \cdot \mathrm{grad} T}{T^2} \right) dt \tag{3}$$

where t_f is the time to failure, W_p is the work due to plasticity, T is the temperature and $\mathbf{J}_q$ is the heat flux due to conduction. The FFE is posed as a constant. Thus, if the FFE for a certain laminate can be estimated, it should be possible to predict the remaining life of the component. However, one needs to measure the temperature variation during loading, which can be difficult in non-laboratory conditions. Additionally, experimental tension-tension fatigue data does only suggest but does not confirm that the FFE is a constant [27]. Furthermore, there does not exist a model to estimate the FFE, which therefore must be obtained from fatigue experiments [25]. Accordingly, this method cannot yet be used for direct prediction of fatigue life a priori.

Considering strategies with damage evolution, Movaghgar and Lvov [30, 31, 32] applied a cycle-by-cycle damage evolution law on the basis of elastic energy W_e, stress ratio R and damage index D:

$$\frac{dD}{dN} = f(D, R, W_e) \tag{4}$$

where $f(D, R, W_e)$ is a suitable function. The model is calibrated with experimental data from reversed cyclic bending experiments on woven glass/epoxy laminates [30]. The resulting function is subsequently used to obtain an analytic expression for prediction of the fatigue life [31]:

$$N_f = \frac{1}{\frac{m}{2^n}(n+1) \cdot \left\{ \sigma_1 \left(\frac{\sigma_1}{E_1} - \frac{\nu_{12}}{E_2}\sigma_2 \right) + \sigma_2 \left(\frac{-\nu_{12}}{E_1}\sigma_1 + \frac{\sigma_2}{E_2} \right) + \frac{\tau_{12}^2}{G_{12}} \right\}^n} \tag{5}$$

In Eq. (5) n and m are fitting parameters, E_i, G_{12} and ν_{12} are the material elastic parameters in there respective planes and σ_i, τ_{12} are the applied multiaxial stresses. The equation was used to predict the fatigue life of a wind turbine blade [31]. In later work [32] the same methodology, although with a different resulting fatigue life equation, was applied to laminates from woven glass/phenolic and woven glass/epoxy in multiple loading directions.

Instead of using a cumulative degradation law, Mao and Mahadevan [33] proposed the use of a direct damage model based on the observed shape of the damage index for composite structures:

$$D = q\left(\frac{n}{N}\right)^{m_1} + (1-q)\left(\frac{n}{N}\right)^{m_2} \tag{6}$$

In Eq. (6) N is the fatigue life at the given load level, n is the amount of cycles and the other variables are fitting parameters [33]. The model predictions were compared to experimental data from tension-tension ($R = 0.1$) fatigue experiments on AS4/PR500 5-harness satin weave composite laminates [34]. A good agreement with test results was obtained. The same model was also used by other authors. Toubal et al. [35] correlated the damage law with experiments for a $[(\pm 45)]_{2s}$ laminate from HR 285/G803 carbon fabric laminae at $R = 0.1$. Montesano et al. [36] modelled the stiffness degradation of off-axis fatigue tests from a two-dimensional 8HS woven carbon fiber-reinforced BMI resin. This investigation also included the determination of the model parameters, Eq. (6), for multiple temperature levels.

Most aforementioned life prediction models do not attempt to account for the actual damage mechanisms but rather consider the laminate ply or layup as a quasi-homogeneous structure. Hereby they assume that the mechanisms can be captured with the use of fitting parameters or damage indices. This approach does not inform the model user about which damage mechanism is dominant for the failure of the structure. Hence he cannot clearly assess which action is best to take to improve his design, which can be a disadvantage. A select number of authors, however, have developed life prediction models based on a consideration of an actual damage mechanism.

Slaughter and Fleck [37] and Dadkhah et al. [38] based their fatigue life prediction models on the mechanism of fibre kink band forming, observed during compression-compression fatigue experiments. They related the geometrical implications of the waviness of the fibres into a mechanism based parameter for the relation of the Whöler curve. Subsequently, a good agreement between predictions and experiments was observed.

In addition to fibre kinking, another well known failure mechanism is ply delamination. Fatigue life models for this type of failure do not generally consider the construction of an S-N curve directly but tend to construct a law for crack growth per fatigue cycle (da/dN). From this law, and the specimen geometry, the final life of the structure can subsequently be calculated. This has, for example, been demonstrated by Chen and Shivakumar et al. [39, 40]. They predicted the fatigue life for mode I Double Cantilever Beam (DCB) experiments based on the total life concept. This concept describes the growth of a crack in all three growth regions; slow, stable and unstable crack growth; using one single equation:

$$\frac{da}{dN} = A\left(\frac{G_{Imax}}{G_{IR}}\right)^m \frac{\left(1-\left(\frac{G_{Ith}}{G_{Imax}}\right)^{D_1}\right)}{\left(1-\left(\frac{G_{Imax}}{G_{IR}}\right)^{D_2}\right)} \tag{7}$$

where a is the crack length, N is the amount of cycles, G_{IR} is the interlaminar fracture resistance, G_{Ith} the mode I interlaminar fracture threshold and G_{Imax} the maximum mode I energy release rate. A, m, D_1 and D_2 are fitting parameters. Apart from an excellent agreement between experiment and prediction, they also performed a sensitivity study on the effect of changing the aforementioned fitting parameters. From which it was concluded that the material parameters and results are quasi-insensitive to changes in the values of the fitting parameters. Many more delamination models exist, many of which are not fatigue life models. To prevent repetition, however, the reader is referred to the recent work of Bak et al. [41] for an overview on fatigue delamination models.

Fatigue life models with a micromechanical basis can be quite useful to predict fatigue life. These methods, however, can only be used to predict the life of laminates where the main failure mechanism is the mechanism the model is designed for. On the one hand, not being careful, one could end up with an incorrect prediction, damaging the specific project progress. On the other hand, the models on a micromechanical basis seem to agree better with experimental results than the models which assume a homogeneous laminate. It is up to the model user to identify the best model for his purpose.

5 Residual Strength Models

Different from the “Fatigue Life Models”, the “Residual Strength Models” attempt to predict residual strength. These methods are discussed in this section. Currently, however, one should refer to, a single residual strength model. Despite an extensive search in the scientific literature, only one model, validated for laminates with three-dimensional fibre architectures, has been found. This model was presented by Post et al. [42, 43].

Post et al. [42] predicted the remaining strength of continuous strand mat and woven quasi-isotropic glass/epoxy specimens subjected to tension-tension fatigue by employing the critical element concept from Reifsnider [44]. In the critical element concept, the plies in the laminate are defined as either critical (their failure will cause failure of the laminate) or non-critical (failure does not immediately cause failure of the laminate). For each critical ply, a remaining strength formulation was constructed dependent on the amount of cycles to failure, the applied stress level per cycle, a fitting parameter and the amount of cycles to failure at the applied stress level. The amount of cycles to failure at the applied stress level was subsequently connected to the measured stiffness degradation during the experiments in order to predict remaining strength. Additionally, using the failure model and the statistical Weibull distribution of the measured fatigue strengths, Monte-Carlo simulations were performed to model the statistical variation in the experiments. Reasonable agreement was obtained for the predicted remaining strengths and statistical variation. Post’s closed form model was extended for variable amplitude loading in 2008 [43]. This was done by truncating the remaining strength phenomenological model from Reifsnider [44] to a finite series:

$$Fr = 1 - \left[\sum_{k=1}^{n} (1-Fa_k)^{\frac{1}{j}} \left(\frac{1}{N(Fa_k)}\right)\right]^j \tag{8}$$

where n is the total amount of cycles, Fa_k is the normalized peak stress for the k-the cycle and $N(Fa_k)$ is the number of cycles to failure at constant amplitude loading. j is a fitting parameter. The number of cycles to failure $N(Fa_k)$ is obtained by fitting a $\log(N) - \log(Fa)$ curve to (constant amplitude) experimental data. Spectrum loading experiments were used for validation. Low-high as well as high-low and random spectra were applied at $10Hz$. It was immediately noticed that the randomized spectrum was much more detrimental than the low-high and high-low. Not surprisingly, overestimation of the life for the randomized spectrum was obtained. The high-low and low-high predictions were satisfactory.

6 Residual Stiffness Models

"Residual Stiffness Models" for textile composites have been presented by a few authors [45, 46, 47, 48]. Apart from predicting failure of the component, these models are also concerned with the prediction of the deformation of the structure during the fatigue process and the resulting stress redistribution.

Early 2001, Khan et al. [45] presented such a model. They related the measured stiffness change from experiments to the damage change per cycle dD/dN through an empirical exponential law. This law was analytically integrated to obtain the fatigue life. The fatigue experiments were performed at a stress ratio $R = 0.1$ and a frequency of 20 Hz on three different stacking sequences from a carbon fabric reinforced polyester resin. For each laminate the empirical parameters were determined and the fatigue life was reasonably well predicted.

Wen and Yazdani [48] presented an anisotropic cumulative damage model for tension-tension fatigue on the basis of the isotropic model developed by Hansen [49]. Damage accumulation was accounted for on the basis of the second invariant of the stress tensor in an exponential equation. A total of three fitting parameters were used. Substituting the damage law into the relation between stress and strain (which is Hooke's law plus a permanent deformation term) resulted in the accumulation model. The model has been applied to the experimental data from [49] on a woven glass/epoxy laminate. From this data also the fitting parameters are derived. As the fitting parameters were derived from the validation data, it is no surprise that a satisfactory agreement with this data was obtained.

Tate and Kelkar [50] proposed a general stiffness reduction approach. The model is essentially a parametric curve based on a fitted S-N curve as input. To fit the S-N curve, a sigmoïdal Boltzmann constant was used. The residual stiffness equation was obtained by integrating the rate of modulus decay. The model was applied to braided carbon/epoxy laminates with a braiding angle of 25, 30 and 45 degrees subjected to uniaxial tension-tension fatigue. The correlation with experimental data was less satisfactory.

The three aforementioned residual stiffness models were presented by their authors as closed form analytic equations. As such they are easy to use and implement, but cannot take into account a change of stress state during loading. Hence, they cannot be applied to variable amplitude loading scenarios, for which a degradative approach is required. An example of this was presented by Van Paepegem and Degrieck [46, 51, 52, 53]. They proposed a damage model that can simulate stiffness damage, stress redistribution and accumulation of permanent strain for fully reversed cyclic bending of woven glass/epoxy laminates. The model has multiaxial loading capability and is validated for both $[(0/90)]_8$ and $[(\pm 45)]_8$ laminates. In the model, a tensor **D** characterizes the amount of damage in each principal element direction. The damage evolution is controlled by a degradation law for each direction which distinguishes between tension and compression loading. Interaction between damage in principle directions is governed through the use of fatigue failure indices. Permanent strain evolution (thought to be caused by the formation of debris under shear loading) is governed by an additional law. The model was incorporated as a material law in finite element simulations which allowed to take into account the effect of stress redistribution due to the material degradation. Good agreement was found between predicted and simulated specimen deformation and applied force (the cyclic bending experiment was displacement controlled).

In addition to Van Paepegem and Degrieck, Hochard et al. [47, 54, 55] have been extensively involved in fatigue damage modelling of woven composite structures. Their research is not only restricted to fatigue but also to the prediction of the behaviour up to static failure [56, 57, 58]. Hochard et al. based their fatigue damage model on a damage model developed for UD carbon/epoxy laminates [59]. The UD model [59] is a combination of static and fatigue laws based on a thermodynamic approach [60]. Additionally, a law for permanent strain development is incorporated. To apply the UD model on a woven ply, Hochard et al. [47, 54] artificially replace the woven ply by two stacked unidirectional plies corresponding to the warp and weft thicknesses. Hence, a (0/90) ply behaves like a $[0_\delta/90_{1-\delta}]$ laminate. δ is the ratio between the warp and weft thicknesses. The woven ply damage tensor can then be defined in function of the UD degradation laws:

$$\begin{cases} d_1 = \frac{(1-\delta) d_2^{*90^\circ} E_2^{0*}}{E_1^0} \\ d_2 = \frac{\delta d_2^{*0^\circ} E_2^{0*}}{E_2^0} \\ d_{12} = \delta d_{12}^{*0^\circ} + (1-\delta) d_{12}^{*90^\circ} \end{cases} \tag{9}$$

where $d_2^{*0^\circ}$, $d_{12}^{*0^\circ}$, $d_2^{*90^\circ}$ and $d_{12}^{*90^\circ}$ are the transverse damage and the shear damage in the unidirectional virtual plies at 0 and 90 degree respectively. E_2^{0*} is the initial transverse stiffness of the two UD* plies. For the UD ply degradation laws, the reader is referred to [54]. A good agreement with experimental data was achieved for unbalanced 5-harness satin weave glass/epoxy on laminates without stress concentrations at different stress levels. The model, however, tended to underestimate the failure strength of laminates with stress concentrations [55] because a local criterion was used to determine final fracture. Hochard et al. [55] proposed to improve this prediction by using, instead of the local criterion, a criterion based on the concept of Fracture Characteristic Volume (FCV). This concept is similar to the point stress

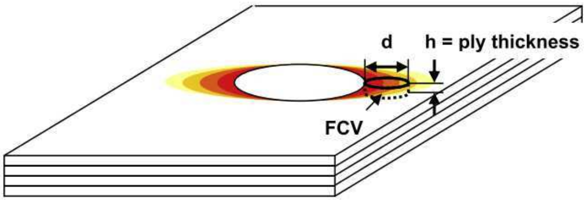

Fig. 6. Hochard et al. [55], non-local failure criterion based on a Fracture Characteristic Volume

and average stress criterion introduced by Whitney and Nuismer [61]. The FCV is a cylinder defined at the ply scale as the volume $V = hS$, where h is equal to the thickness of the ply and S is the in-plane area, see Figure 6. The non-local failure criterion was subsequently defined in terms of the thermodynamic energy contained in this volume. From initial experiments on laminates from unbalanced woven e-glass/epoxy plies, it was observed that using the FCV improved the failure prediction for static as well as fatigue load. Although the predictions were improved, as with the point and average stress criterion, the size of the FCV has to be determined using experimental observations and can differ depending on the load scenario and characteristics of the material. More validation work needs to be done to assess this drawback properly.

7 Mechanistic Models

In this section the mechanism based models for fatigue damage in textile FRP are presented. As aforementioned, "Mechanistic Models" are damage models that attempt to model the actual damage mechanisms which occur during fatigue loading. Note, however, that a physical representation of the damage in the analysis is not the only way to model damage mechanisms. The occurrence of damage can also be represented by an equivalent stiffness or strength degradation. Damage modelling with equivalent methods, however, is generally performed at the micro- or mesoscale because of convergence issues in macroscale simulations. Therefore, modelling on smaller scales does not generally satisfy the need for the assessment of the behavioural changes at the component level. To resolve this, micro- and mesoscale analyses are often coupled to the macroscale, which results in multiscale models.

Indeed, no less than three different length scales are brought together in this section. However, the different scales (micro, meso, macro) are not clearly defined in the scientific literature, which resulted in the mixed use of the terminology. Nonetheless, a definition of the three scales is formulated below. The microscale is defined as the scale on which the individual fibre filaments and matrix are treated separately while the mesoscale distinguishes between the fibre yarns and matrix. The multiscale models combine the models on the different scales to allow more accurate predictions. The smaller scales (micro or meso), however, always form the basis of the models. In the following paragraphs, these models are presented.

Huang [62] predicted the fatigue life of woven fabric composites based on the unit cell approach. In this approach, a representative element is constructed from the periodic fibre architecture and then analyzed as a subproblem. In earlier works [63,64], Huang had already constructed a bridging model for the analysis of such a unit cell. The model has mostly been used for static failure prediction [65,66,67] but in 2002 it was also applied for fatigue [62]. The simulation involved predicting the residual strength, which was obtained by using S-N curves of the constituents, fibre and matrix. The predictions correlated reasonably well with measured S-N curves. For better predictions, it was suggested to use exact constituent properties and fabric geometry parameters.

A stiffness degradation model, which employs macro degradation with its roots in the microscale, has been presented by Gude et al. [68] to model damage evolution in 3D knitted glass/epoxy reinforced laminates. Gude et al. modified the degradation model of Van Paepegem and Degrieck [46] by replacing the dependency of the cycle-by-cycle damage tensor from stress and damage to material effort and damage:

$$\frac{d\mathbf{D}}{dn} = f(\sigma_{ij}, \mathbf{D}) \Longrightarrow \frac{d\mathbf{D}}{dn} = f(E_{ff}, \mathbf{D}) \tag{10}$$

The material effort E_{ff} is calculated from the microscale failure model by Cuntze [69] and used in the degradation law as follows:

$$\left.\frac{dD}{dn}\right|_{0^\circ} = a_1 e^{a_2 E_{ff}^{||\sigma}} \tag{11a}$$

$$\left.\frac{dD}{dn}\right|_{90^\circ} = a_3 e^{E_{ff}^{\perp\sigma,2}} (D_{s2} - D_2) \tag{11b}$$

In Eq.(11) a_1, a_2, a_3 and D_{s2} are model parameters. D_2 is the damage parameter in the 2-direction with respect to the ply primary axis system. The model predictions were compared to the stiffness degradation and residual strength obtained from tension-tension fatigue experiments. A good correlation with this data was obtained.

After gaining experience with closed form fatigue degradation [36], Montesano et al. [70, 71] more closely observed the damage occurring during tension-tension ($R = 0.1$) fatigue loading of triaxially braided carbon-epoxy laminates and discovered that the observed stiffness loss is directly related to the crack density during loading [70]. On the basis of this discovery, they used an exponential fitting of the measured crack densities for both braider yarns and interface and linked this to the macroscopic stiffness degradation through a cumulative degradation factor. Correlation with experimental observations was excellent. The model proposed, however, is not complete as the measured damage growth was obtained by fitting a curve. To be fully mechanism based, the fitting parameters should be related to the applied loading.

Gagel et al. [72] presented a similar approach for the prediction of fatigue in non-crimp fabric reinforced epoxy laminates. Here, also the crack densities were obtained from experiments and associated with the ply damage vector. The interaction of the several cracks in different directions was subsequently modeled using a 3D finite element analysis. Comparison of the predictions to tension-compression experiments resulted in a reasonable agreement in stiffness loss over cycles.

Xu [73, 74] presented an algorithm for modelling the mesoscale progressive fatigue damage and to predict the fatigue lifetime and fatigue modulus in textile-reinforced composites. A single layer of a twill weave carbon/epoxy resin was modelled as a unit cell in FEA. First, separate failure criteria for the yarns and matrix were established to simulate progressive static failure. Static failure, only considered for the homogeneous, transversely isotropic yarns of the unit cell, was determined using the Tsai-Wu criterion. Subsequently, the type of failure was determined and the stiffness of the failed element reduced according to the rules used by Zako et al. [75]. Inter-yarn matrix failure is not considered.

The quasi-static damage model was subsequently extended for fatigue damage using a cycle by cycle approach. Using this approach, a predefined number of cycles is simulated by first loading the unit cell to the maximum stress using the quasi-static model and letting the failing elements degrade statically. Then, the stiffness of the elements is degraded due to fatigue for the number of cycles using S-N curves, the Palmgren-Miner rule and the anisotropic damage model by Liu et al. [76]. As a last step, the unit cell is unloaded and reloaded while allowing simultaneous degradation of all material points. Allowing interaction between the material points should restore the disturbed equilibrium state from the previous step. The predictions by the model were compared to the stiffness reduction and S-N curves of tension-tension fatigue experiments on the aforementioned material. The agreement with S-N curves was satisfactory. The agreement with the stiffness degradation was less satisfying since the model predicted a 10% stiffness degradation while this was not observed in the experiments.

It is possible that the cause of this discrepancy lies with the use of the Tsai-Wu criterion for the prediction of failure. From the World Wide Failure Exercise (WWFE) [9] , it is known that most of the current failure criteria cannot accurately predict failure in all quadrants of the failure envelope. Specifically for the Tsai-Wu criterion, the strength in the biaxial compression quadrant tends to be overestimated, while it tends to be underestimated in the biaxial compression-shear quadrant. Therefore, it is plausible that, during the highly multiaxial stress analysis of the elements in the RUC, the moment of failure initiation was incorrectly estimated which lead to a premature drop in laminate stiffness. Therefore, a well chosen criterion for failure initiation can be quite important. Currently, however, there is no consensus on which failure criterion is best to use because virtually all current models have disadvantages. Consequently, it is up to the researcher to select the most appropriate criterion for his/her problem. Since static failure criteria are not the topic of this discussion, the reader is referred to the results from the WWFE [9] and a more recent review on static failure prediction methods by Icardi et al. [7].

Similar to Xu et al., Min et al. [77] presented an analytical micro-meso-macro approach for fabric reinforced ceramic composites. The fabric structure was modeled and the resulting stiffness computed using the homogenization technique presented by Naik [78]. The yarn damage was modeled through the integration of the three-phase micromechanics, the shear-lag and the continuum fracture mechanics model. The strength parameters, used as input for the yarn model, were determined using a statistical model based on a Weibull strength distribution function. The degradation of the matrix material between the yarns was modeled using a phenomenological stiffness degradation law. Nevertheless the analytical nature of the model, a reasonable agreement with experimental data was obtained. An indication that analytical methods can still be of value for highly complex failure problems.

Gu et al. [79, 80, 81] have presented mesoscale analyses of 3D angle-interlock woven and 3D braided textile structures subjected to cyclic three-point bending. The yarn structure and matrix were modelled as separate entities in an

FEA model of the entire bending specimen. The FEA model used three-dimensional solid elements and the stiffness degradation of fibre yarns and matrix were modelled separately using degradative laws. Yarn-matrix interface delamination was modeled using a damage initiation and Paris delamination growth equation. Although it was shown that the model is able to describe failure locations of the yarns and matrix [82], no comparisons with experimental results have been presented yet. Therefore, it is not possible to evaluate this method properly.

8 Discussion

In the previous Sections the existing fatigue modelling techniques for textile composites were presented. In search for new insights for further development of the modelling techniques, a discussion is now provided on their capabilities and a comparison to techniques developed for laminates from UD laminae.

Investigating the models provided for each of the modelling categories (Section 4 to Section 7) it is observed that the models presented in closed form are most common. This is because they are easy to apply and implement, and allow for fast predictions. Therefore these models are very suitable for parametric analysis and quick estimations for conceptual and preliminary design. The same trend is observed for UD FRP. Just recently, Kawai et al. [83] applied the "CFL-diagram approach" (Section 4) to unidirectional carbon/epoxy laminates. The same technique used to construct the fatigue diagrams for the laminates from woven plies could be applied. They did have to divide the diagram in four instead of two separate segments to catch outlying data points. Other recent models for UD FRP were presented by Wu and Yao [84] and Liu and Mahadevan [76]. Wu and Yao [84] used a fitted empirical stiffness degradation parameter for damage characterization. Liu and Mahadevan [76], similar to the aforementioned work of Naderi and Khonsari [25], used a multiaxial fatigue failure criterion based on previous work in metals. The quadratic criterion relies on an equivalent stress derived from the multiaxial stress state. Acceptable agreement with experimental data was found.

Although the mentioned works are fairly recent, their basis has been laid some time ago. In 1981, Hashin [85] already tried to predict multiaxial fatigue failure based on quadratic stress failure criteria which used S-N curves as input for the fatigue strengths. Many authors as Shokrieh and Lessard [86], who proposed a multiaxial residual strength model, and Yao and Himmel [87], who used a cumulative damage law with a closed form residual strength curve for variable amplitude fatigue, have followed since. The focus of these models has been to improve correlation with engineering and design curves. As such, most of these models depend on a great number of experimental data, which restricts their applicability to the laminates and loading types for which they have been validated. Therefore, these curves lose part of their value when the structural behaviour of an actual component subjected to non-conventional loading over time needs to be predicted.

In Section 5, only one approach has been presented which predicts residual strength. The final model for variable amplitude [43] was an expansion of the closed form model [42] to predict residual strength. For FRP consisting of UD lamina, more residual strength models exist, [88, 8, 89]. Note, however, that since 2008 no pure residual strength models have been presented; not for laminates from UD, nor laminates from woven plies. The reduced interest in these models results from the increased interest in residual stiffness models which are capable of predicting the behaviour of the structure during fatigueing. Inherently, residual strength models do not possess this capability. For this purpose, residual stiffness models are more suitable.

It is for this reason that residual stiffness approaches have been gaining popularity. The deformation of a laminate can be derived from the analysis at any stage of the fatigue life, which is quite valuable. A few Stiffness degradation approaches were presented (Section 6). The approaches discussed allow predictions for multiaxial fatigue loading and permanent strain accumulation. The agreement between model prediction and experimental observation ranges from good to moderate. This is an indication that well chosen relationships can be very useful, even if they incorporate some fitting parameters. Care should be taken, however. Fitting parameters do not possess a physical meaning and therefore have to be determined using dedicated experiments. Because of this, an abundance of fitting parameters could require an experimental program equally extensive as that required for some of the "Fatigue Life Models" (Section 4). On top of that, the extendability of the model for scenarios other than the ones calibrated for, could be questioned.

The first "Residual Stiffness Models" were used to predict stiffness degradation on uniaxially loaded specimens. Later, it was recognized that these models could also be used as a material law (on the ply level) in FEA analysis [46, 90]. This allowed the analysis of more complex structures which exhibit stress redistribution during fatigue loading. The technique has recently been applied to laminates from UD laminae by Eliopoulos et al. [91, 92] and Lian et al. [93]. Eliopoulos et al. [91, 92] modelled stiffness and strength degradation and applied these as a material model in the FEA simulation of glass/epoxy UD test coupons at various stress ratios. The model consists of both a strength and a stiffness degradation scheme. The stiffness degradation was used to simulate degradation due to fatigueing, while the strength degradation triggered, using Puck's failure criterion, a "sudden death" of the material element which was simulated with a sudden stiffness degradation in the element. The predictions agreed well with simulations. Lian et al. [93] performed a similar exercise but used the Hashin failure criteria and different stiffness and strength degradation curves. The FEA simulation was performed with 3D brick elements instead of the shell elements used by Eliopoulos et al. [91, 92]. The validation, however, was only done using one stress ratio. Nevertheless, good agreement with experimental data was shown.

Another notable contribution was presented by Varvani-Farahani and Shirazi [94,95]. They proposed a fatigue damage model based on considerations of the failure mechanisms in a UD ply. By using the rule of mixtures, a parameter for the fibre-matrix interface as well as a dependence on the off-axis angle for a single ply was introduced in a cumulative damage law which only depends on the amount of cycles to failure. Additionally, a weighing factor was used for the individual contributions of the 0 and 90 degree plies in the [0/90] laminate [94]. The methodology was further validated for [$0/\theta$] laminates [95]. A good agreement with experiments was found. This is remarkable since only 2 experimental parameters, the amount of cycles to failure at a given stress ratio and the weighing factor for the contributions of the plies, need to be estimated. It should be noted, however, that the model has only been validated for uniaxial fatigue experiments, although different stress ratios were tested.

The model by Varvani-Farahani and Shirazi [94,95] is close to a mechanistic model as it is loosely based on micromechanics. In Section 7 the mechanistic models for textile composites found in the scientific literature have been presented. A total of 6 models were discussed from which all but one have been published within the last 5 years. This indicates that this modelling technique has not yet reached maturity. The technique, however, is not new. Since the introduction of composites for high performance structural applications, researchers have attempted to predict failure and stiffness based on micromechanical considerations. Nowadays these basic theories can be found in standard works on composite design [96,97]. The available experimental, analytic and computational techniques at that time, however, did not allow for a thorough analysis and identification of the occurring damage mechanisms.

Regarding damage mechanisms, in the past years Quaresimin and Carraro et al. [98,99,100,101] have performed an in-depth investigation of the aforementioned in UD laminae subjected to multiaxial loads. Their first investigations [98,99] were meticulous measurements and analyses of matrix crack initiation and propagation rates in tubular specimens from UD glass/epoxy laminae subjected to tension-torsion loading. A similar exercise [100] followed on flat specimens subjected to uniaxial tension-tension fatigue, where the laminate layup produced a similar internal multiaxial stress field. The results from the uniaxial tests agreed well with the results from the tubular specimens, and it was confirmed that the evolution of intraply damage is basically the same when the internal multiaxial stress in the specimens is similar. This is an interesting result for characterization of composite materials and their response to a multiaxial stress state. In the latest work [101] they moved from experiments to modelling and proposed a new Finite Fracture Mechanics (FFM) model for the initiation of cracks at the fibre-matrix interface. Comparison of the model to experiments was promising. The effect of the microcracks on the response of the structure at the macroscale was not yet considered. However, it is an important issue for the prediction of the behaviour of a component size structure.

Nowadays, computational power is reaching the stage where mapping the effects of damage on the small scale to the mechanical response on the large scale is feasible. Examples of multiscale models already exist for static loading by Ernst et al. [102], Zhou et al. [103,104] and Šmilauer et al. [105]. These models use the concept of a Repetitive Unit Cell (RUC) of the fibre-matrix or yarn-matrix structure, with separate failure criteria for the aforementioned, to accurately predict the progressive failure of the laminate due to the textile architecture. These (static) models compare well with experimental data but can suffer from mesh dependency as illustrated by Rolfes et al. [106]. For fatigue, 2 multiscale models, Yang et al. [79] and Xu [73], were presented in the previous section. The model by Yang et al. [79], however, does not make use of a RUC but explicitly models the yarns and fibres in the entire specimen and Xu's [73] model needs improvement to more accurately predict the modulus loss. Further developments are expected in the future.

To aid research, the current multiscale fatigue damage models for FRP from UD laminae could used as inspiration. Shokrieh and Lessard [107,108], for example, proposed a generalized micromechanics model for UD lamina which incorporates strength and stiffness degradation with a failure analysis linked to "sudden-death". The model was used to predict failure in unaxial tension-tension and compression-compression tests as well as the fatigue behaviour of pin-bolted joints. Similar models were more recently presented by Kennedy et al. [109] and Passipoularidis et al. [110]. In the model by Shokrieh and Lessard, an analytic micromechanics model was formulated. One could also separate the fibre and matrix contributions by modelling them as individual constituents in, for example, an FEA analysis. This was done by Qian et al. [111,112], among others.

An issue with the latest modelling technique is the significant need for computational resources. Microscale FEA models are generally low cost but a large number of these computations must be solved for mapping to the macroscale. Researchers have sought ways to improve these computation speeds, which resulted in three ways of multiscale modelling: "Full" (FE^2), "Concurrent" and "Hierarchical". In the "FE^2" approach, the analysis is performed to the full extent, e.g. in every integration point of the macroscale model resides a micromodel which is solved simultaneously. This is also the most computationally intensive approach. In the "Concurrent" approach only critical integration points of the macroscale model contain the micromodel. This results in decreased computation cost but might also decrease the accuracy of the model. In the "Hierarchical" approach the micromodel is analyzed before the analysis of the macromodel. The results are stored and subsequently used in the macroscale model. Violeau et al. [113] call this approach a "handbook"-type and estimate that, with respect to the "FE^2" approach, the calculation time could be reduced by a factor 100-1000.

Another approach is the Synergistic Damage Mechanics (SDM) concept introduced by Talreja et al. [114, 115, 116]. In the approach the effect of microscale damage (e.g. cracks) is related to continuum damage mechanics (CDM) using the constraint parameter κ. κ is therefore a measure for the ability of damage to change the overall material response. It is directly related to the Crack Opening Displacement (COD). By performing dedicated analysis on the microscale with micromechanics [115] as well as FEA [114], κ was predetermined for several damage states of cross-ply UD laminates and subsequently used to predict stiffness for a given crack density. Good agreement with experiments was found. In subsequent work, the stiffness prediction for a given damage was expanded to model damage evolution [117] using the critical energy concept. An increase of damage was estimated when the work on the specimen passed a critical level. The SDM was used to predict static failure for the World Wide Failure Exercise - Part III (WWFE-III) [116]. No comparison with experimental data from this exercise has yet been published. Although the SDM has only been applied to predict static failure, it has potential for predicting fatigue failure. To allow this, the damage evolution laws should be modified to include the effect of cyclic loading.

In addition to computational efficiency, another key element for the academic and commercial success of a model is the accuracy of the prediction when supplied with a minimum amount of experimental data. Many models struggle with predicting for unknown materials and unknown loading [8]. This is especially true for the “Fatigue Life”, “Residual Strength” and “Residual Stiffness Models” since they are not based on actual damage mechanisms. The “Mechanistic Models”, and particularly the multiscale models, attempt to address this issue. Their promise to be more generally applicable is very attractive to both researchers and industry. This will cause more mechanistic models to emerge and be improved over the next few years. The (future) success of these models, however, depends on whether they can obtain these projected capabilities and consequently reduce cost while increasing efficiency for composite testing and design.

9 Conclusion

Manufacturers of high volume products do not have the means nor the time to perform extensive mechanical testing to put a new product to the market. Therefore, these manufacturers extensively rely on simulation to predict product performance. Driven by the promise of energy savings and increased structural efficiency, automotive industries are currently considering to introduce textile (carbon and glass) composite structures into primary load carrying components. The simulation of these materials for fatigue loading, however, is not yet quite perfect. Over the past years, several developments in modelling techniques for fatigue damage assessment have been presented in the scientific literature. In this work, the capabilities of the existing models for textile FRP have been reviewed and compared to models for UD FRP.

To accommodate the newly born models and changes in terminology, the classification of Degrieck and Van Paepegem [5] was modified to four categories; (i) “Fatigue Life Models”, (ii) “Residual Strength Models”, (iii) “Residual Stiffness Models” and (iv) “Mechanistic Models”. The existing models were presented in their respective categories and their capabilities assessed. From the discussion, the following conclusions are made:

a) “Fatigue Life Models” and closed form "Residual Strength and Stiffness Models"’ are very suitable for fast predictions and parametric studies but rely on a large amount of experimental data.
b) “Residual Strength Models” are receiving a reduced amount of interest due to the growing demand for the prediction of mechanical properties while loading.
c) “Residual Stiffness Models” are very suitable for the prediction of the mechanical performance, but generally suffer from the need for experimental data to determine fitting parameters.
d) “Residual Stiffness Models” can be combined with a residual strength approach.
e) “Mechanistic models” attempt to model the actual damage mechanisms occurring during fatigue. This type of modelling has roots in the early days of composites, but nowadays the computational and experimental resources are available to perform detailed analyses, predictions and multiscale simulations.
f) “Mechanistic Models” are currently not mature enough for commercial use. However, their promise of general applicability makes them very attractive for further development.

Acknowledgements

The author acknowledges the financial support from Strategic Initiative Materials in Flanders (SIM) through the “SBO1 M3Strength” Project.

Historical Perspective on Classification Modifications

This appendix provides a historical perspective on the evolution of language concerning the naming of fatigue damage modelling techniques. In Section 3, the two reasons to modify the existing classification were given as:

1. The usage of the terminology has changed

2. Mechanistic modelling strategies are more pronounced

Concerning {1}, Degrieck and Van Paepegem [5] divided the modelling techniques into: (i) fatigue life models, (ii) phenomenological models for residual stiffness/strength and (iii) progressive damage models. While the definition of fatigue life models, the prediction of remaining fatigue life on the basis of S-N curves, has not changed, the definitions of phenomenological and progressive have. In the original definition, phenomenological models predict the experimentally observable change in macroscopic properties, e.g. residual strength or stiffness, of a component. These models consisted of a few expressions which could predict the remaining properties under a certain loading and amount of cycles. The progressive damage models were set as models that contain one or more damage variables related to measurable manifestations of damage such as transverse matrix cracking or delamination size. A weakness of the term “progressive”, however, is that it can also be used as the definition for a “gradual” approach. Since no more appropriate term exists it should come as no surprise that, when the phenomenological models were extended for gradual degradation [91, 102, 110], the term “progressive” became used in a phenomenological context. The usage of this term has thus lost the connection with the measurable manifestations of damage.

Concerning {2}, with the promise for generally applicable approaches, several researchers started to consider modelling frameworks based on the mechanics of the constituents of the laminate. Analytical micromechanical models for the prediction of (static) failure prediction and stiffness already existed (e.g. Puck failure criterion, rule of mixtures, etc... [118,78,96]). Models to predict the pristine macroscopic laminate stiffness through homogenization of a segmented representation of the yarn structure also existed. An overview of these methods has recently been presented by Hallal et al. [119]. Specific examples are the work from Kregers and Melbardis [120], and Whitney and Chou [121].

In 1999, Ladevèze [122] was one of the first to investigate the path of including damage in the micro/mesomodel before the homogenization step. This developed into a new research field over the past years [123, 124, 125]. Ultimately, not only the micro and macroscale were considered but also the intermediate mesoscale; particularly important for FRP with three-dimensional architectures [102]. When the paper by Degrieck and Van Paepegem [5] was published, this modelling technique had only just been proposed and could therefore not have been considered for fatigue. Nowadays, this area is becoming a distinguishable research field.

To accommodate the developments in research and terminology, two modifications to the classification by Degrieck and Van Paepegem [5] are proposed. First, since the term “progressive” can now be interpreted in multiple ways, it is replaced with “Mechanistic”. In this way the specific modelling approach contained in this class is made more clear. Second, “Residual Strength” and “‘Residual Stiffness” models are separated and the term “phenomenological” is omitted. This results in the classes “Residual Strength Models” and “Residual Stiffness Models”. The category “Fatigue Life Models” remains unchanged.

With the proposed changes, the classification now consists of four categories: (i) “Fatigue Life Models”, (ii) “Residual Strength Models”, (iii) “Residual Stiffness Models” and (iv) “Mechanistic Models”. For a detailed definition of these categories, the reader is referred to Section 3.

References

[1] Ayranci, C., and Carey, J., 2008. “2D braided composites: A review for stiffness critical applications”. *Composite Structures,* **85**(1), Sept., pp. 43–58.

[2] Ladevèze, P., 2004. “Multiscale modelling and computational strategies for composites”. *International Journal for Numerical Methods in Engineering,* **60**(1), May, pp. 233–253.

[3] McCartney, L., 2008. “Energy methods for fatigue damage modelling of laminates”. *Composites Science and Technology,* **68**(13), Oct., pp. 2601–2615.

[4] Zhang, C., and Xu, X., 2013. “Finite element analysis of 3D braided composites based on three unit-cells models”. *Composite Structures,* **98**, Apr., pp. 130–142.

[5] Degrieck, J., and Van Paepegem, W., 2001. “Fatigue damage modeling of fibre-reinforced composite materials: Review”. *Applied Mechanics Reviews,* **54**(4), p. 279.

[6] Pascoe, J., Alderliesten, R., and Benedictus, R., 2013. “Methods for the prediction of fatigue delamination growth in composites and adhesive bonds. A critical review”. *Engineering Fracture Mechanics,* **112-113**, Nov., pp. 72–96.

[7] Icardi, U., Locatto, S., and Longo, A., 2007. “Assessment of Recent Theories for Predicting Failure of Composite Laminates”. *Applied Mechanics Reviews,* **60**(2), p. 76.

[8] Post, N. L., Case, S. W., and Lesko, J. J., 2008. “Modeling the variable amplitude fatigue of composite materials: A review and evaluation of the state of the art for spectrum loading”. *International Journal of Fatigue,* **30**(12), Dec., pp. 2064–2086.

[9] Soden, P., Kaddour, A., and Hinton, M., 2004. “Recommendations for designers and researchers resulting from the world-wide failure exercise”. *Composites Science and Technology,* **64**(3–4), Mar., pp. 589–604.

[10] Quaresimin, M., Susmel, L., and Talreja, R., 2010. “Fatigue behaviour and life assessment of composite laminates under multiaxial loadings”. *International Journal of Fatigue,* **32**(1), Jan., pp. 2–16.

[11] Liu, P., and Zheng, J., 2010. “Recent developments on damage modeling and finite element analysis for composite laminates: A review”. *Materials & Design,* **31**(8), Sept., pp. 3825–3834.

[12] Daggumati, S., De Baere, I., Van Paepegem, W., Degrieck, J., Xu, J., Lomov, S., and Verpoest, I., 2013. “Fatigue and post-fatigue stressâĂŞstrain analysis of a 5-harness satin weave carbon fibre reinforced composite”. *Composites Science and Technology,* **74**, Jan., pp. 20–27.

[13] Cox, B., Dadkhah, M., and Inman, R., 1992. “Mechanisms of compressive failure in 3D composites”. *Acta Metallurgica et Materialia,* **40**(12), pp. 3285–3298.

[14] Kawai, M., and Matsuda, Y., 2012. “Anisomorphic constant fatigue life diagrams for a woven fabric carbon/epoxy laminate at different temperatures”. *Composites Part A: Applied Science and Manufacturing,* **43**(4), Apr., pp. 647–657.

[15] Kawai, M., Matsuda, Y., and Yoshimura, R., 2012. “A general method for predicting temperature-dependent anisomorphic constant fatigue life diagram for a woven fabric carbon/epoxy laminate”. *Composites Part A: Applied Science and Manufacturing,* **43**(6), June, pp. 915–925.

[16] Kawai, M., and Taniguchi, T., 2006. “Off-axis fatigue behavior of plain weave carbon/epoxy fabric laminates at room and high temperatures and its mechanical modeling”. *Composites Part A: Applied Science and Manufacturing,* **37**(2), Feb., pp. 243–256.

[17] Kawai, M., and Murata, T., 2010. “A three-segment anisomorphic constant life diagram for the fatigue of symmetric angle-ply carbon/epoxy laminates at room temperature”. *Composites Part A: Applied Science and Manufacturing,* **41**(10), Oct., pp. 1498–1510.

[18] Vania, A., and Carvelli, V., 2010. *Fitting Approach of the Fatigue Tensile Response of Textile Composite Materials.* Destech Publications, Inc, Lancaster.

[19] Carvelli, V., Gramellini, G., Lomov, S. V., Bogdanovich, A. E., Mungalov, D. D., and Verpoest, I., 2010. “Fatigue behavior of non-crimp 3D orthogonal weave and multi-layer plain weave E-glass reinforced composites”. *Composites Science and Technology,* **70**(14), Nov., pp. 2068–2076.

[20] Mouritz, A. P., 2005. “A Simple Fatigue Life Model for Three-dimensional Fiber-Polymer Composites”. *Journal of Composite Materials,* **40**(5), July, pp. 455–469.

[21] Toumi, R. B., Renard, J., Monin, M., and Nimdum, P., 2013. “Fatigue Damage Modelling of Continuous E-glass Fibre/Epoxy Composite”. *Procedia Engineering,* **66**, pp. 723–736.

[22] Tamuzs, V., Dzelzitis, K., and Reifsnider, K., 2008. “Prediction of the cyclic durability of woven composite laminates”. *Composites Science and Technology,* **68**(13), Oct., pp. 2717–2721.

[23] Tamuzs, V., Dzelzitis, K., and Reifsnider, K., 2004. “Fatigue of Woven Composite Laminates in Off-Axis Loading I. The Mastercurves”. *Applied Composite Materials,* **11**(5), Sept., pp. 259–279.

[24] Tamuzs, V., Dzelzitis, K., and Reifsnider, K., 2004. “Fatigue of Woven Composite Laminates in Off-Axis Loading II. Prediction of the Cyclic Durability”. *Applied Composite Materials,* **11**(5), Sept., pp. 281–293.

[25] Naderi, M., and Khonsari, M. M., 2011. “A comprehensive fatigue failure criterion based on thermodynamic approach”. *Journal of Composite Materials,* **46**(4), Oct., pp. 437–447.

[26] Naderi, M., Kahirdeh, A., and Khonsari, M. M., 2012. “Dissipated thermal energy and damage evolution of Glass/Epoxy using infrared thermography and acoustic emission”. *Composites Part B: Engineering,* **43**(3), Apr., pp. 1613–1620.

[27] Naderi, M., and Khonsari, M., 2012. “Thermodynamic analysis of fatigue failure in a composite laminate”. *Mechanics of Materials,* **46**, Mar., pp. 113–122.

[28] Naderi, M., and Khonsari, M., 2013. “On the role of damage energy in the fatigue degradation characterization of a composite laminate”. *Composites Part B: Engineering,* **45**(1), Feb., pp. 528–537.

[29] Naderi, M., Amiri, M., and Khonsari, M. M., 2009. “On the thermodynamic entropy of fatigue fracture”. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences,* **466**(2114), Oct., pp. 423–438.

[30] Movaghghar, A., and Lvov, G. I., 2011. “An Energy Model for Fatigue Life Prediction of Composite Materials Using Continuum Damage Mechanics”. *Applied Mechanics and Materials,* **110-116**, Oct., pp. 1353–1360.

[31] Movaghghar, A., and Lvov, G. I., 2011. “A Method of Estimating Wind Turbine Blade Fatigue Life and Damage using Continuum Damage Mechanics”. *International Journal of Damage Mechanics,* **21**(6), Sept., pp. 810–821.

[32] Movaghghar, A., and Lvov, G. I., 2012. “Theoretical and Experimental Study of Fatigue Strength of Plain Woven Glass/Epoxy Composite”. *Journal of Mechanical Engineering,* **58**(3), Mar., pp. 175–182.

[33] Mao, H., and Mahadevan, S., 2002. “Fatigue damage modelling of composite materials”. *Composite Structures,* **58**(4), Dec., pp. 405–410.

[34] Kumar, R., and Talreja, R., 2000. “Fatigue damage evolution in woven fabric composites”. In Collection of the 41st Aiaa/Asme/Asce/Ahs/Asc Structures, Structural Dynamics, and Materials Conference and Exhibit, Vol 1 Pts 1-3, pp. 1841–1849.

[35] Toubal, L., Karama, M., and Lorrain, B., 2006. “Damage evolution and infrared thermography in woven composite laminates under fatigue loading”. *International Journal of Fatigue,* **28**(12), Dec., pp. 1867–1872.

[36] Montesano, J., Selezneva, M., Fawaz, Z., Poon, C., and Behdinan, K., 2012. “Elevated temperature off-axis fatigue behavior of an eight-harness satin woven carbon-fiber/bismaleimide laminate”. *Composites Part A: Applied Science and Manufacturing,* **43**(9), Sept., pp. 1454–1466.

[37] Slaughter, W., and Fleck, N., 1993. “Compressive Fatigue of Fibre Composites”. *Journal of the Mechanics and Physics of Solids,* **41**(8), pp. 1265–1284.

[38] Dadkhah, M., Cox, B., and Morris, W., 1995. “Compression-Compression Fatigue of 3D Woven Composites”. *Acta Metallurgica,* **43**(12), pp. 4235–4245.

[39] Chen, H., Shivakumar, K., and Abali, F., 2006. “A comparison of total fatigue life models for composite laminates”. *Fatigue & Fracture of Engineering Materials & Structures,* **29**(1), Jan., pp. 31–39.

[40] Shivakumar, K., Chen, H., Abali, F., Le, D., and Davis, C., 2006. “A total fatigue life model for mode I delaminated composite laminates”. *International Journal of Fatigue,* **28**(1), Jan., pp. 33–42.

[41] Bak, B. L. V., Sarrado, C., Turon, A., and Costa, J., 2014. “Delamination Under Fatigue Loads in Composite Laminates : A Review on the Observed Phenomenology and Computational Methods”. *Applied Mechanics Reviews,* **66**(November).

[42] Post, N., Bausano, J., Case, S., and Lesko, J., 2006. “Modeling the remaining strength of structural composite materials subjected to fatigue”. *International Journal of Fatigue,* **28**(10), Oct., pp. 1100–1108.

[43] Post, N. L., Cain, J., McDonald, K. J., Case, S. W., and Lesko, J. J., 2008. “Residual strength prediction of composite materials: Random spectrum loading”. *Engineering Fracture Mechanics,* **75**(9), June, pp. 2707–2724.

[44] Reifsnider, K. L., and Case, S. W., 2002. *Damage tolerance and durability of material systems.* Wiley Interscience.

[45] Khan, Z., Al-Sulaiman, F. A., Farooqi, J. K., and Younas, M., 2001. “Fatigue Life Predictions in Woven Carbon Fabric/Polyester Composites Based on Modulus Degradation”. *Journal of Reinforced Plastics and Composites,* **20**(5), Mar., pp. 377–398.

[46] Van Paepegem, W., and Degrieck, J., 2005. “Simulating damage and permanent strain in composites under in-plane fatigue loading”. *Computers & Structures,* **83**(23-24), Sept., pp. 1930–1942.

[47] Hochard, C., Payan, J., and Bordreuil, C., 2006. “A progressive first ply failure model for woven ply CFRP laminates under static and fatigue loads”. *International Journal of Fatigue,* **28**(10), Oct., pp. 1270–1276.

[48] Wen, C., and Yazdani, S., 2008. “Anisotropic damage model for woven fabric composites during tensionâĂŞtension fatigue”. *Composite Structures,* **82**(1), Jan., pp. 127–131.

[49] Hansen, U., 1999. “Damage Development in Woven Fabric Composites during Tension-Tension Fatigue”. *Journal of Composite Materials,* **33**(7), Apr., pp. 614–639.

[50] Tate, J. S., and Kelkar, A. D., 2008. “Stiffness degradation model for biaxial braided composites under fatigue loading”. *Composites Part B: Engineering,* **39**(3), Apr., pp. 548–555.

[51] Van Paepegem, W., and Degrieck, J., 2002. “A new coupled approach of residual stiffness and strength for fatigue of fibre-reinforced composites”. *International Journal of Fatigue,* **24**, pp. 747–762.

[52] Van Paepegem, W., and Degrieck, J., 2003. “Modelling damage and permanent strain in fibre-reinforced composites under in-plane fatigue loading”. *Composites Science and Technology,* **63**, pp. 677–694.

[53] Van Paepegem, W., and Degrieck, J., 2004. “Simulating in-plane fatigue damage in woven glass fibre-reinforced composites subject to fully reversed cyclic loading”. *Fatigue & Fracture of Engineering Materials & Structures,* **27**(July), pp. 1197–1208.

[54] Hochard, C., and Thollon, Y., 2010. “A generalized damage model for woven ply laminates under static and fatigue loading conditions”. *International Journal of Fatigue,* **32**(1), Jan., pp. 158–165.

[55] Hochard, C., Miot, S., and Thollon, Y., 2014. “Fatigue of laminated composite structures with stress concentrations”. *Composites Part B: Engineering,* **65**, Oct., pp. 11–16.

[56] Bordreuil, C., and Hochard, C., 2004. “Finite Element Computation of Woven Ply Laminated Composite Structures up to Rupture”. *Applied Composite Materials,* **11**(3), May, pp. 127–143.

[57] Thollon, Y., and Hochard, C., 2009. “A general damage model for woven fabric composite laminates up to first failure”. *Mechanics of Materials,* **41**(7), July, pp. 820–827.

[58] Benamira, M., Hochard, C., and Haiahem, A., 2011. “Behaviour to failure of fibre mat reinforced composite under combined loading conditions”. *Composites Part B: Engineering,* **42**(6), Sept., pp. 1412–1419.

[59] Payan, J., and Hochard, C., 2002. “Damage Modelling of Laminated Carbon/Epoxy Composites Under Static and Fatigue Loadings”. *International journal of f,* **24**, pp. 299–306.

[60] Ladeveze, P., and LeDantec, E., 1992. “Damage modelling of the elementary ply for laminated composites”. *Composites Science and Technology,* **43**(3), pp. 257–267.

[61] Whitney, J., and Nuismer, R., 1974. “Stress Fracture Criteria for Laminated Composites Containing Stress Concentrations”. *Journal of Composite Materials,* **8**(3), Jan., pp. 253–265.

[62] Huang, Z.-M., 2002. “Fatigue Life Prediction of a Woven Fabric Composite Subjected to Biaxial Cyclic Loads”.

Composites Part A: Applied Science and Manufacturing, **33**(2), Feb., pp. 253–266.

[63] Huang, Z.-M., 2000. "A Unified Micromechanical Model for the Mechanical Properties of Two Constituent Composite Materials Part I: Elastic Behavior". *Journal of Thermoplastic Composite Materials,* **13**(4), pp. 252–271.

[64] Huang, Z.-M., 2001. "Micromechanical Prediction of Ultimate Strength of Transversely Isotropic Fibrous Composites". *International journal of solids and structures,* **38**, pp. 4147–4172.

[65] Huang, Z.-M., 2005. "Efficient Approach to the Structure-Property Relationship of Woven and Braided Fabric-reinforced Composites up to Failure". *Journal of Reinforced Plastics and Composites,* **24**(12), Aug., pp. 1289–1309.

[66] Huang, Z.-M., 2004. "A Bridging Model Prediction of the Ultimate Strength of Composite Laminates Subjected to Biaxial Loads". *Composites Science and Technology,* **64**(3–4), Mar., pp. 395–448.

[67] Huang, Z.-M., and Liu, L., 2014. "Predicting strength of fibrous laminates under triaxial loads only upon independently measured constituent properties". *International Journal of Mechanical Sciences,* **79**, Feb., pp. 105–129.

[68] Gude, M., Hufenbach, W., and Koch, I., 2010. "Damage evolution of novel 3D textile-reinforced composites under fatigue loading conditions". *Composites Science and Technology,* **70**(1), Jan., pp. 186–192.

[69] Cuntze, R., 1997. *Neue Bruchkriterien und Festigkeitsnachweise f{ü}r unidirektionalen Faserkunststoffverbund unter mehrachsiger Beanspruchung: Modellbildung und Experimente ; BMBF-F{ö}rderkennzeichen 03N8002 ; Abschlu{ß}bericht 1997.* 5]: [Fortschritt-Berichte VDI. VDI-Verlag.

[70] Montesano, J., Fawaz, Z., Behdinan, K., and Poon, C., 2013. "Fatigue damage characterization and modeling of a triaxially braided polymer matrix composite at elevated temperatures". *Composite Structures,* **101**, July, pp. 129–137.

[71] Montesano, J., Bougherara, H., and Fawaz, Z., 2014. "Application of infrared thermography for the characterization of damage in braided carbon fiber reinforced polymer matrix composites". *Composites Part B: Engineering,* **60**, Apr., pp. 137–143.

[72] Gagel, A., Fiedler, B., and Schulte, K., 2006. "On modelling the mechanical degradation of fatigue loaded glass-fibre non-crimp fabric reinforced epoxy laminates". *Composites Science and Technology,* **66**(5), May, pp. 657–664.

[73] Xu, J., 2011. "Meso-Scale Finite Element Fatigue Modelling of Textile Composite Materials". PhD thesis, KU Leuven.

[74] Xu, J., Lomov, S. V., Verpoest, I., Daggumati, S., Paepegem, W. V., Degrieck, J., and Olave, M., 2013. "A progressive damage model of textile composites on meso-scale using finite element method: static damage analysis". *Journal of Composite Materials,* **48**(25), Oct., pp. 3091–3109.

[75] Zako, M., Uetsuji, Y., and Kurashiki, T., 2003. "Finite element analysis of damaged woven fabric composite materials". *Composites Science and Technology,* **63**(3-4), Feb., pp. 507–516.

[76] Liu, Y., and Mahadevan, S., 2007. "A unified multiaxial fatigue damage model for isotropic and anisotropic materials". *International Journal of Fatigue,* **29**(2), Feb., pp. 347–359.

[77] Min, J., Xue, D., and Shi, Y., 2014. "Micromechanics modeling for fatigue damage analysis designed for fabric reinforced ceramic matrix composites". *Composite Structures,* **111**, May, pp. 213–223.

[78] Naik, A., 1994. Analysis of Woven and Braided Fabric Reinforced Composites. Tech. rep., Analytical Services & Materials, Inc., Virginia.

[79] Yang, G., Sun, B., and Gu, B., 2014. "Large-scale finite element analysis of a 3D angle-interlock woven composite undergoing low-cyclic three-point bending fatigue". *Journal of the Textile Institute,* **105**(3), Mar., pp. 275–293.

[80] Wu, L., Zhang, F., Sun, B., and Gu, B., 2014. "Finite element analyses on three-point low-cyclic bending fatigue of 3-D braided composite materials at microstructure level". *International Journal of Mechanical Sciences,* **84**, July, pp. 41–53.

[81] Wu, L., Sun, B., and Gu, B., 2014. "Numerical analyses of bending fatigue of four-step three-dimensional rectangular-braided composite materials from unit cell approach". *The Journal of The Textile Institute,* **106**(1), Mar., pp. 67–79.

[82] Wu, L., and Gu, B., 2014. "Fatigue behaviors of four-step three-dimensional braided composite material: a meso-scale approach computation". *Textile Research Journal,* **84**(18), July, pp. 1915–1930.

[83] Kawai, M., and Itoh, N., 2014. "A failure-mode based anisomorphic constant life diagram for a unidirectional carbon/epoxy laminate under off-axis fatigue loading at room temperature". *Journal of Composite Materials,* **48**(5), Feb., pp. 571–592.

[84] Wu, F., and Yao, W., 2010. "A fatigue damage model of composite materials". *International Journal of Fatigue,* **32**(1), Jan., pp. 134–138.

[85] Hashin, Z., 1981. "Fatigue failure criteria for unidirectional fiber composites". *Journal of Applied Mechanics,* **48**(December 1981), pp. 846–852.

[86] Shokrieh, M., and Lessard, L., 1997. "Multiaxial fatigue behaviour of unidirectional plies based on uniaxial fatigue experiments-I. Modelling". *International Journal of Fatigue,* **19**(3), pp. 201–207.

[87] Yao, W., and Himmel, N., 2000. "A new cumulative fatigue damage model for fibre-reinforced plastics". *Composites Science and Technology,* **60**, pp. 1–6.

[88] Philippidis, T., and Passipoularidis, V., 2007. "Residual strength after fatigue in composites: Theory vs. experiment".

International Journal of Fatigue, **29**(12), Dec., pp. 2104–2116.

[89] Reifsnider, K., Case, S., and Duthoit, J., 2000. "The mechanics of composite strength evolution". *Composites Science and Technology,* **60**, pp. 2539–2546.

[90] Papanikos, P., Tserpes, K. I., and Pantelakis, S., 2003. "Modelling of fatigue damage progression and life of CFRP laminates". *Fatigue & Fracture of Engineering Materials and Structures,* **26**(1), Jan., pp. 37–47.

[91] Eliopoulos, E. N., and Philippidis, T. P., 2011. "A progressive damage simulation algorithm for GFRP composites under cyclic loading. Part I: Material constitutive model". *Composites Science and Technology,* **71**(5), Mar., pp. 742–749.

[92] Eliopoulos, E. N., and Philippidis, T. P., 2011. "A progressive damage simulation algorithm for GFRP composites under cyclic loading. Part II: FE implementation and model validation". *Composites Science and Technology,* **71**(5), Mar., pp. 750–757.

[93] Lian, W., and Yao, W., 2010. "Fatigue life prediction of composite laminates by FEA simulation method". *International Journal of Fatigue,* **32**(1), Jan., pp. 123–133.

[94] Varvani-Farahani, A., and Shirazi, A., 2007. "A Fatigue Damage Model for (0/90) FRP Composites based on Stiffness Degradation of 0 and 90 Composite Plies". *Journal of Reinforced Plastics and Composites,* **26**(13), June, pp. 1319–1336.

[95] Shirazi, A., and Varvani-Farahani, A., 2009. "A Stiffness Degradation Based Fatigue Damage Model for FRP Composites of $(0/\theta)$ Laminate Systems". *Applied Composite Materials,* **17**(2), Aug., pp. 137–150.

[96] Daniel, I., and Ishai, O., 2006. *Engineering mechanics of composite materials*, second ed. Oxford University Press, New York.

[97] Kassapoglou, C., 2010. *Design and Analysis of Composite Structures.* John Wiley & Sons, Ltd, Chichester, UK, Sept.

[98] Quaresimin, M., and Carraro, P., 2013. "On the investigation of the biaxial fatigue behaviour of unidirectional composites". *Composites Part B: Engineering,* **54**, Nov., pp. 200–208.

[99] Quaresimin, M., and Carraro, P., 2014. "Damage initiation and evolution in glass/epoxy tubes subjected to combined tension-torsion fatigue loading". *International Journal of Fatigue*, Jan., pp. 1–11.

[100] Quaresimin, M., Carraro, P., Mikkelsen, L., Lucato, N., Vivian, L., Brondsted, P., Sorensen, B., Varna, J., and Talreja, R., 2014. "Damage evolution under cyclic multiaxial stress state: A comparative analysis between glass/epoxy laminates and tubes". *Composites Part B: Engineering,* **61**, May, pp. 282–290.

[101] Carraro, P., and Quaresimin, M., 2014. "Modelling fibre-matrix debonding under biaxial loading". *Composites Part A: Applied Science and Manufacturing,* **61**, June, pp. 33–42.

[102] Ernst, G., Vogler, M., Hühne, C., and Rolfes, R., 2010. "Multiscale Progressive Failure Analysis of Textile Composites". *Composites Science and Technology,* **70**(1), Jan., pp. 61–72.

[103] Zhou, Y., Lu, Z., and Yang, Z., 2013. "Progressive damage analysis and strength prediction of 2D plain weave composites". *Composites Part B: Engineering,* **47**, Apr., pp. 220–229.

[104] Lu, Z., Zhou, Y., Yang, Z., and Liu, Q., 2013. "Multi-scale finite element analysis of 2.5D woven fabric composites under on-axis and off-axis tension". *Computational Materials Science,* **79**, Nov., pp. 485–494.

[105] Šmilauer, V., Hoover, C. G., Bažant, Z. P., Caner, F. C., Waas, A. M., and Shahwan, K. W., 2011. "Multiscale simulation of fracture of braided composites via repetitive unit cells". *Engineering Fracture Mechanics,* **78**(6), Apr., pp. 901–918.

[106] Rolfes, R., Vogler, M., Czichon, S., and Ernst, G., 2011. "Exploiting the structural reserve of textile composite structures by progressive failure analysis using a new orthotropic failure criterion". *Computers & Structures,* **89**(11-12), June, pp. 1214–1223.

[107] Shokrieh, M. M., and Lessard, L. B., 2000. "Progressive Fatigue Damage Modeling of Composite Materials, Part I: Modeling". *Journal of Composite Materials,* **34**(13), July, pp. 1056–1080.

[108] Shokrieh, M. M., and Lessard, L. B., 2000. "Progressive Fatigue Damage Modeling of Composite Materials, Part II: Material Characterization and Model Verification". *Journal of Composite Materials,* **34**(13), July, pp. 1081–1116.

[109] Kennedy, C. R., Brádaigh, C. M. O., and Leen, S. B., 2013. "A multiaxial fatigue damage model for fibre reinforced polymer composites". *Composite Structures,* **106**, Dec., pp. 201–210.

[110] Passipoularidis, V., Philippidis, T., and Brondsted, P., 2011. "Fatigue life prediction in composites using progressive damage modelling under block and spectrum loading". *International Journal of Fatigue,* **33**(2), Feb., pp. 132–144.

[111] Qian, C., Westphal, T., Kassapoglou, C., and Nijssen, R., 2013. "Development of a multi-fibre unit cell for use in modelling of fatigue of unidirectional composites". *Composite Structures,* **99**, May, pp. 288–295.

[112] Qian, C., Westphal, T., and Nijssen, R., 2013. "Micro-mechanical fatigue modelling of unidirectional glass fibre reinforced polymer composites". *Computational Materials Science,* **69**, Mar., pp. 62–72.

[113] Violeau, D., Ladevèze, P., and Lubineau, G., 2009. "Micromodel-based simulations for laminated composites". *Composites Science and Technology,* **69**(9), July, pp. 1364–1371.

[114] Singh, C. V., and Talreja, R., 2009. "A synergistic damage mechanics approach for composite laminates with matrix cracks in multiple orientations". *Mechanics of Materials,* **41**(8), Aug., pp. 954–968.

[115] Varna, J., and Talreja, R., 2012. "Integration of Macro-and MicroDamage Mechanics for the Performance Evalua-

tion of Composite Materials". *Mechanics of Composite Materials,* **48**(2), May, pp. 145–160.

[116] Singh, C. V., and Talreja, R., 2012. "A synergistic damage mechanics approach to mechanical response of composite laminates with ply cracks". *Journal of Composite Materials,* **47**(20-21), Dec., pp. 2475–2501.

[117] Singh, C. V., and Talreja, R., 2010. "Evolution of ply cracks in multidirectional composite laminates". *International Journal of Solids and Structures,* **47**(10), May, pp. 1338–1349.

[118] Puck, A., and Schürmann, H., 1998. "Failure Analysis of FRP Laminates by Means of Physically based Phenomenological Models". *Composites Science and Technology,* **58**(7), July, pp. 1045–1067.

[119] Hallal, A., Younes, R., and Fardoun, F., 2013. "Review and comparative study of analytical modeling for the elastic properties of textile composites". *Composites Part B: Engineering,* **50**, July, pp. 22–31.

[120] Kregers, A. F., and Melbardis, Y., 1978. "Determination of the deformability of three-dimensionally reinforced composites by the stiffness averaging method". *Polymer Mechanics,* **14**(1), pp. 1–5.

[121] Whitney, T. J., and Chou, T.-W., 1989. "Modeling of 3-D Angle-Interlock Textile Structural Composites". *Journal of Composite Materials,* **23**(9), Sept., pp. 890–911.

[122] Ladevèze, P., and Dureisseix, D., 1999. "Une nouvelle stratégie de calcul micro/macro en mécanique des structures". *Comptes Rendus de l'Académie des Sciences- ...*, pp. 1237–1244.

[123] Xia, Z., Chen, Y., and Ellyin, F., 2000. "A meso / micro-mechanical model for damage progression in glass- fiber / epoxy cross-ply laminates by finite-element analysis". *Composites Science and Technology,* **60**, pp. 1171–1179.

[124] Ladeveze, P., 2005. "A Bridge Between the Micro-and Mesomechanics of Laminates: Fantasy or Reality?". *Mechanics of the 21st Century*, pp. 187–201.

[125] Talreja, R., 2008. "Damage and fatigue in composites - A personal account". *Composites Science and Technology,* **68**(13), Oct., pp. 2585–2591.

List of Tables

List of Figures